$$\lim_{x\to 3} \frac{x^2-x-6}{4-\sqrt{5x+1}} = \lim_{x\to 3} \frac{(x-3)(x+2)}{4-\sqrt{5x+1}}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+2)}{4-\sqrt{5x+1}} \cdot \frac{4+\sqrt{5x+1}}{4+\sqrt{5x+1}}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+2)(4+\sqrt{5x+1})}{16-(5x+1)}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+2)(4+\sqrt{5x+1})}{-5x+15}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)(4+\sqrt{5x+1})}{-5\cancel{(x-3)}}$$

$$= \lim_{x\to 3} \frac{(x+2)(4+\sqrt{5x+1})}{-5}$$

$$= \frac{(3+2)(4+\sqrt{5.3+1})}{-5}$$

$$= \frac{5(4+4)}{\cancel{-5}\ -1}$$

$$= -8$$

5) $$\lim_{x\to 6} \frac{\sqrt{3x-2}-\sqrt{2x+4}}{x-6}$$

$$= \lim_{x\to 6} \frac{\sqrt{3x-2}-\sqrt{2x+4}}{x-6} \cdot \frac{\sqrt{3x-2}+\sqrt{2x+4}}{\sqrt{3x-2}+\sqrt{2x+4}}$$

$$= \lim_{x\to 6} \frac{(3x-2)-(2x+4)}{(x-6)(\sqrt{3x-2}+\sqrt{2x+4})}$$

$$\cancel{= \lim_{x\to 6} \frac{(3x-2)^2-(2x+4)^2}{(x-6)^2(\sqrt{3x-2}+\sqrt{2x+4})^2}}$$

$$= \lim_{x\to 6} \frac{\cancel{(x-6)}}{\cancel{(x-6)}(\sqrt{3x-2}+\sqrt{2x+4})}$$

$$= \frac{1}{\sqrt{3(6)-2}+\sqrt{2(6)+4}}$$

$$= \frac{1}{\sqrt{16}+\sqrt{16}} = \frac{1}{4+4} = \frac{1}{8}$$

(3) $$\lim_{x\to 0} \frac{4x}{\sqrt{1-2x}-\sqrt{1+2x}}$$

$$= \lim_{x\to 0} \frac{4x}{\sqrt{1-2x}-\sqrt{1+2x}} \cdot \frac{\sqrt{1-2x}+\sqrt{1+2x}}{\sqrt{1-2x}+\sqrt{1+2x}}$$

$$= \lim_{x\to 0} \frac{4x(\sqrt{1-2x}+\sqrt{1+2x})}{(1-2x)-(1+2x)}$$

$$= \lim_{x\to 0} \frac{\cancel{4x}(\sqrt{1-2x}+\sqrt{1+2x})}{-\cancel{4x}}$$

$$= \lim_{x\to 0} -(\sqrt{1-2x}+\sqrt{1+2x})$$

$$= -(\sqrt{1-2(0)}+\sqrt{1+2.0})$$

$$= -2$$

(4) $$\lim_{x\to \infty} \sqrt{x(x+5)} - 2x + 1 \ldots$$

$$= \lim_{x\to \infty} \sqrt{x(x+5)}-(2x-1) \cdot \frac{\sqrt{x(x+5)}+(2x-1)}{\sqrt{x(x+5)}-(2x-1)}$$

$$= \lim_{x\to \infty} \frac{(x(x+5))-(2x-1)^2}{\sqrt{x(x+5)}-(2x-1)}$$

$$= \lim_{x\to \infty} \frac{(x^2+5x)-(4x^2-4x+1)}{\sqrt{x(x+5)}-(2x-1)}$$

$$= \lim_{x\to \infty} \frac{x^2+5x-4x^2+4x-1}{\sqrt{x(x+5)}-(2x-1)}$$

$$= \lim_{x\to \infty} \frac{-3x^2+9x-1}{\sqrt{x(x+5)}-2x+1}$$

Luc

① $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ = \left(2\cos\left(\frac{40+80}{2}\right)\cos\left(\frac{40-80}{2}\right)\right) + \cos 1$

$\sin(-x) = -\sin x$

$\boxed{\cos(-x) = \cos x}$

$\sin(-90) = -\sin 90$

$\cos(-60) = \cos$

$= \left(2\cos 60^\circ \cos(-20^\circ)\right) + \cos 160^\circ$

$= \left(\cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \cos(-20)\right) + \cos 160^\circ$

$2 \cdot \cos\left(\frac{-20+160}{2}\right)\cos\left(\frac{-20-160}{2}\right)$

$= 2 \cdot \cos 70^\circ \; \cos -90^\circ = 0$

$= 2 \cdot \cos 70^\circ \cdot \cancel{\cos 70} 1$

$= 2\cos 70$

$= 0$

② $\sin 105^\circ + \cos 15^\circ = \sin(45+60)^\circ + \cos(45^\circ - 30^\circ)$

$= \left(\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ\right) + \left[\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 3\right.$

$= \left(\left(\frac{1}{2}\sqrt{2}\right)\cdot\left(\frac{1}{2}\right) + \left(\frac{1}{2}\sqrt{2}\right)\left(\frac{1}{2}\sqrt{3}\right)\right) + \left[\left(\frac{1}{2}\sqrt{2}\right)\left(\frac{1}{2}\sqrt{3}\right) + \left(\frac{1}{2}\sqrt{2}\right)\left(\frac{1}{2}\right.\right.$

$= \left[\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6}\right] + \left[\frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}\right]$

$= \frac{\sqrt{2}+\sqrt{6}}{4} + \frac{\sqrt{2}+\sqrt{6}}{4}$

$= \frac{2\sqrt{2}+2\sqrt{6}}{4} = \frac{2(\sqrt{2}+\sqrt{6})}{4} = \frac{1}{2}(\sqrt{2}+\sqrt{6})$

③ $\cos 165^\circ = \cos(135+30)^\circ$

$= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$

$= \left[\left(-\frac{1}{2}\sqrt{2}\right)\cdot\left(\frac{1}{2}\sqrt{3}\right)\right] - \left[\left(\frac{1}{2}\sqrt{2}\right)\left(\frac{1}{2}\right)\right]$

$= \left(-\frac{1}{4}\sqrt{6}\right) - \frac{1}{4}\sqrt{2}$

$= \frac{-\sqrt{6}-\sqrt{2}}{4} = -\frac{1}{4}(\sqrt{6}+\sqrt{2})$

④ $0 < x < \pi, x = ?$

$\cos 2x + \cos x = 0$

$(2\cos^2 x - 1) + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2} \;\vee\; \cos x = -1$

$x = \arccos \frac{1}{2} = 60^\circ$

$x = \arccos -1 = 180^\circ$

1) $f(x) = \sin^2(2x + \pi/6)$

$f'(0) = \ldots ?$

$f'(0) = [\cos^2(2x+\pi/6)]\ 2$

$= \ldots$

$f(x) = \sin^2(\ldots + \frac{\pi}{\ldots})$

$= 2\sin(2x + \frac{\pi}{6}) \cdot \cos(2x + \frac{\pi}{6})\ (2)$

$= 4\sin(2x + \frac{\pi}{6}) \cos(2x + \frac{\pi}{6})$

$= 2 \cdot 2\sin(2x + \frac{\pi}{6}) \cos(2x + \frac{\pi}{6})$

$= 2\sin(4x + \ldots)$

$= 2\sin(4 \cdot 0 + \frac{\pi}{3})$

$= 2\sin\frac{\pi}{3}$

$= 2\sin 60^\circ = 2 \cdot \frac{1}{2}\sqrt{3} \cdot \sqrt{3}$

$\pi = 180^\circ$

① $f(x) = \sin^2(2x + \pi/6)$

$f'(0) = 2\sin(2x + \pi/6) \cdot \cos(2x + \pi/6)$

$= 2 \cdot \sin(2(2x + \pi/6))$

$= 2 \cdot \sin(4x + \frac{\pi}{3})$

$= 2\sin(4 \cdot (0) + \frac{\pi}{3})$

$= 2\sin\frac{\pi}{3}$

$= 2\sin 60^\circ$

$= 2 \cdot \frac{1}{2}\sqrt{3}$

$= \sqrt{3}$

② $f(x) = \sin^4(3x^2 - 2)$

$f'(x) = 4\sin^3(3x^2 - 2) \cdot \cos(3x^2 - 2)(6x)$

$= 4\sin^3(3x^2 - 2) \cdot \cos(3x^2 - 2) \cdot 6x$

$= 12x \cdot \sin^2(\ldots)$

③ $f(x) = \sqrt[3]{\cos^2(3x^2 + 5x)}$

~~$f'(x) = 3\sqrt[2]{\cos^2(3x^2 + 5x)}$~~

$f'(x) = 3\sqrt[2]{\cos^2(3x^2+5x)} \cdot 2\cos(3x^2+5x) \cdot -\sin(3x^2+5x)$

$= 3\sqrt{\cos^2(3x^2+5x)} \cdot 2\cos(3x^2+5x)$

$f(x) = \left[\cos^2(3x^2+5x)\right]^{\frac{1}{3}}$

$= \frac{1}{3}\left[\cos^5(3x^2+5x)\right]^{-2/3} \cdot 5\cos^4(3x^2+5x)$

$-\sin(3x^2+5x) \cdot (6x+5)$

$=$

$\cos^5(3x^2+5x)$

$= \left[\cos(3x^2+5)\right]^5$

$5\left[\cos 3x^2+5\right]^4$

$-\sin(3x^2+5)$

$5\cos^4(3x^2+5)(-\sin$

$$\lim_{x\to 0} \frac{\cdots}{3x^2-12x+12}$$

$$= \cancel{\lim_{x\to 0} \frac{-(\cos^2(x-2)-1)}{}}$$

$$= \lim_{x\to 0} \frac{1-\cos^2(x-2)}{(3x-6)(x-2)}$$

$$\lim_{x\to 0} \frac{1-(\cos(x-2)\cos(x-2))}{(3x-6)(x-2)}$$

$$= \lim_{x\to 0} \frac{1-(\cos(x-2)\cos(x-2))}{3(x-2)(x-2)}$$

$$= \lim_{x\to 0} \frac{1}{(3x-6)(x-2)} - \lim \frac{\cos(x-2)\cos(x-2)}{3(x-2)^2}$$

$$= \frac{1}{12} - \lim_{x\to 0} \frac{1}{3}\left(\frac{\cos(x-2)}{(x-2)}\right)^2$$

$$= \frac{1}{12} - \frac{1}{3} \cdot (1)^2$$

$$= \frac{1}{12} - \frac{1}{3}$$

$$= \frac{1-4}{12} = \frac{-3}{12} = -\frac{1}{4}$$

(14) $$\lim_{x\to 0} \frac{x-\pi}{2(x-\pi)+\tan(x-\pi)}$$

$$= \lim_{x\to 0} \frac{(x-\pi)}{2(x-\pi)+\tan(x-\pi)}$$

$$= \lim_{x\to 0} \frac{1}{\frac{2(x-\pi)+\tan(x-\pi)}{(x-\pi)}}$$

$$= \frac{1}{2+1} = \frac{1}{3}$$

(15) $$\lim_{x\to \frac{\pi}{2}} \frac{\cos 3x - \cos x}{\sin 2x \cdot \cos 2x}$$

(16) $$\lim_{x\to 0} \frac{4x^2}{1-\cos 2x}$$

$$= \lim_{x\to 0} \frac{4x^2}{1-(1-2\sin^2 x)}$$

$$= \lim_{x\to 0} \frac{4x^2}{2\sin^2 x}$$

$$= \frac{4}{2} \cdot \lim_{x\to 0} \left(\frac{x}{\sin x}\right)^2$$

$$= 2 \cdot (1)^2$$

$$= 2$$

(17) $$\lim_{x\to 0} \frac{\sin 2x}{3-\sqrt{2x+9}} \cdot \frac{3+\sqrt{2x+9}}{3+\sqrt{2x+9}}$$

$$= \lim_{x\to 0} \frac{\sin 2x\,(3+\sqrt{2x+9})}{9-(2x+9)}$$

$$= \lim_{x\to 0} \frac{\sin 2x\,(3+\sqrt{2x+9})}{-2x}$$

$$= \frac{1}{-2} \cdot (2) \lim_{x\to 0} (3+\sqrt{2x+9})$$

$$= -1\,(3+\sqrt{2\cdot(0)+9})$$

$$= -6$$

$$\lim_{x\to 2}\left[\frac{\quad}{x^2-4}\cdot\frac{\quad}{x-2}\right]$$

$$=\lim_{x\to 2}\frac{2-x}{(x+2)(x-2)}\cdot\frac{1}{(x-2)}$$

$$=\lim_{x\to 2}\frac{-\cancel{(x-2)}}{(x+2)(x-2)\cancel{(x-2)}}$$

$$=\cancel{\lim_{x\to 2}\frac{1}{(x+2)^2}}\quad\frac{-1}{(2+2)(2-2)}$$

$$=\cancel{\frac{\cdot 1}{(2+4)^2}}\quad -\frac{1}{4}$$

$$\lim_{x\to 0}\frac{3x}{\sqrt{9+x}-\sqrt{9-x}}$$

$$\lim_{x\to 0}\frac{3x}{\sqrt{9+x}-\sqrt{9-x}}\cdot\frac{\sqrt{9+x}+\sqrt{9-x}}{\sqrt{9+x}+\sqrt{9-x}}$$

$$\lim_{x\to 0}\frac{3x(\sqrt{9+x}+\sqrt{9-x})}{(9+x)-(9-x)}$$

$$\lim_{x\to 0}\frac{3\cancel{x}(\sqrt{9+x}+\sqrt{9-x})}{2\cancel{x}}$$

$$\lim_{x\to 0}\frac{3}{2}(\sqrt{9+x}+\sqrt{9-x})$$

$$\frac{3}{2}(\sqrt{9+0}+\sqrt{9-0})$$

$$=\frac{3}{\cancel{2}}\cdot\cancel{(6)}\,3=9$$

$$\lim_{(y-2)\to 0}\frac{1}{y-2}\left(\frac{1}{2y^2-y-3}-\frac{2}{y^2+y}\right)$$

$$\lim_{(y-2)\to 0}\frac{1}{y-2}\left(\frac{1}{(2y-3)(y+1)}-\frac{2}{y^2+y}\right)$$

$$\frac{1}{y-2}\left[\frac{1}{(2y-3)(y+1)}-\frac{2}{y(y+1)}\right]$$

⑧ $$\lim_{x\to\infty}\sqrt{x+5}+\sqrt{2x-1}\cdot\frac{\sqrt{x+5}\ \ \sqrt{2x-1}}{\sqrt{x+5}-\sqrt{2x-1}}$$

$$=\lim_{x\to\infty}\frac{(x+5)-(2x-1)}{(\sqrt{x+5}-\sqrt{2x-1})}$$

$$=\lim_{x\to\infty}\frac{-x+6}{\sqrt{x+5}-\sqrt{2x-1}}$$ dibagi dg pangkat tertinggi: x

$$=\lim_{x\to\infty}\frac{-\frac{x}{x}+\frac{6}{x}}{\sqrt{\frac{x}{x^2}+\frac{5}{x}}-\sqrt{\frac{2x}{x^2}-1}}$$

$$=\frac{-1+0}{\sqrt{0}-\sqrt{0}}=\frac{-1}{0}=\infty$$

⑨ $$\lim_{x\to 0}\frac{x^2}{1-\sqrt{1+x^2}}\cdot\frac{1+\sqrt{1+x^2}}{1+\sqrt{1+x^2}}$$

$$=\lim_{x\to 0}\frac{x^2(1+\sqrt{1+x^2})}{1-(1+x^2)}$$

$$=\lim_{x\to 0}\frac{\cancel{x^2}(1+\sqrt{1+x^2})}{\cancel{x^2}}$$

$$=1+\sqrt{1+0}$$

$$=2$$

⑩ $$\lim_{x\to 0}\frac{1-\cos 2x}{x\tan\frac{1}{2}x}$$

$$=\lim_{x\to 0}\frac{1-(2\cos^2x-1)}{x\pm\sqrt{\frac{1-\cos x}{1+\cos x}}}$$

$$=\lim_{x\to 0}\frac{2-2\cos^2x}{x\pm\sqrt{\frac{1-\cos x}{1+\cos x}}}$$

$$=\lim_{x\to 0}\frac{2(1-\cos^2x)}{x\pm}$$